M. Math. Algebra I

Mid-Term Examination

September 14, 2005

Instructions. All questions carry equal marks.

1. Prove that, in any group, the intersection of two subgroups of finite index is a subgroup of finite index. (Hint: first prove that any subgroup of finite index contains a normal subgroup of finite index.)

2. Show that there is no non-trivial group homomorphism from the additive group of rational numbers to a finite group.

3. Let A_n denote the (alternating) group of even permutations on n letters. Prove or disprove: Every finite group G is a subgroup of A_n for some n.

4. Define class equation of a finite group G. Determine the class equation of a non-abelian group of order pq, where p and q are (necessarily distinct!) primes.

5. Let G, H be finite groups and $\phi : G \to H$ be a surjective group homomorphism. Prove or disprove: Image of a Sylow subgroup of G is a Sylow subgroup of H.

6. Let G be a finite group and P be a Sylow subgroup of G. Let H be a subgroup of G containing the normalizer of P. Prove that H is its own normalizer.

7. Prove that S_5, the permutation group on 5 letters, does not contain any subgroup of order 15.

8. Let G be a abelian group of order $10,000$. Prove that there exists a surjective group homomorphism from G onto $Z/2Z$. Generalize this question to the maximum extent you can and prove that generalization.